

An experimental study of turbulent convective heat transfer from a flat plate

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A turbulent boundary layer developing on a smooth heated uniform-temperature plate in a zero pressure gradient was set up. The origins of the layers were matched to remove the effect of an unheated starting length. Similarity proposals were tested. The mean flow field followed the usual law of the wall and defect law for both temperature and velocity. Broad-band measurements of streamwise velocity and temperature fluctuations were made, and wall similarity and Townsend's self-preserving flow similarity were found to be applicable, at least after a sufficient flow development.

Some initial attempts to arrive at a comparison between heat and momentum transport are presented. The results include conditionally sampled measurements of instantaneous heat and momentum fluxes and correlations between these two quantities. The fluxes were divided into quadrants. Conditional probabilities and weighted joint probability density functions were measured to determine whether there was a similarity in behaviour of these two fluxes. The concept of 'hole size' developed for momentum flux was extended to heat flux and events corresponding to bursts and sweeps in the momentum flux were found to be accompanied by corresponding events in the heat flux.

1. Introduction

A two-dimensional turbulent boundary layer developing on a smooth heated surface in a zero pressure gradient without an unheated starting length provides one of the simplest flows in turbulent heat-transfer studies: see, for example, the review paper by Kestin & Richardson (1963). It is also one of the few flows where the Reynolds analogy or its various modifications might be valid since the mean boundary conditions for the velocity and temperature fields are identical. The properties of the mean velocity field have been well established for some time (see, for example, Coles & Hirst 1968), and it is now accepted that similar correlation schemes such as a thermal law of the wall and a thermal defect law (see, for example, Reynolds, Kays & Kline 1958; Perry, Bell & Joubert 1966; Brundrett *et al.* 1965; Kader & Yaglom 1972) also exist for the mean temperature profiles. However, it appears to the authors that there has been very little systematic effort applied to deducing scaling laws for the fluctuations in streamwise velocity, and in particular, the temperature fluctuations.

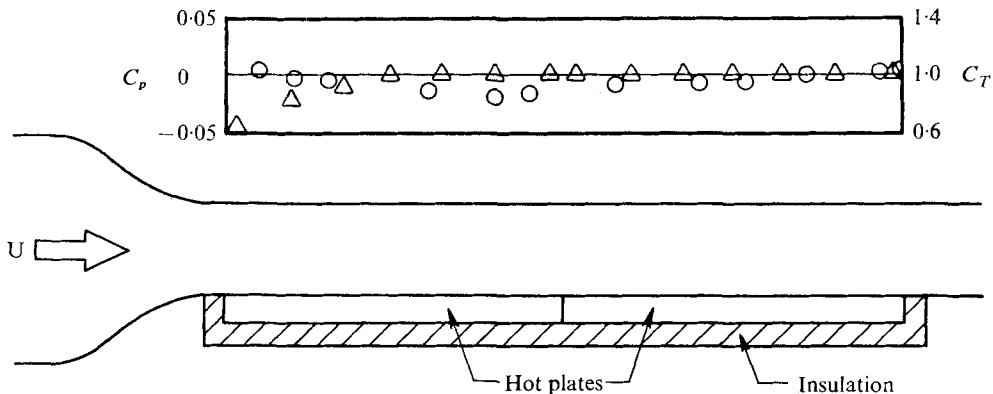


FIGURE 1. Schematic diagram of working section and static pressure and plate temperature coefficients. \circ , C_p ; \triangle , C_T .

In contrast, much work has been done lately on the structure of turbulence, in particular, the structure and generation of Reynolds stress near the wall; see, for example, Kim, Kline & Reynolds (1971), Corino & Brodkey (1969), Kline *et al.* (1967), Willmarth & Lu (1972), Laufer & Badri Narayanan (1970) and Rao, Narashimha & Badri Narayanan (1971). All of this work, however, has been for isothermal flows, and there appears to be no work available in the literature which has studied the statistical properties of the instantaneous Reynolds stress and compared it with the instantaneous heat flux.

This paper covers work with the following aims.

(i) To study a zero-pressure-gradient turbulent boundary layer on a heated constant-temperature wall to determine the scaling laws for the mean and fluctuating temperatures and velocities.

(ii) To make a preliminary study of some statistical properties of the Reynolds stress and heat flux in such a layer.

2. Experimental apparatus and procedure

Wind tunnel

The experiments were carried out in a closed-return wind tunnel at the University of Melbourne. The working section (90×40 cm at the inlet) was fitted with a flexible roof to facilitate the adjustment of the pressure gradient, and the free-stream turbulence level was approximately 0.3%. A schematic diagram of the arrangement is shown in figure 1.

Two separate, identically heated aluminium plates 76.2 cm wide and 152.4 cm long were used, placed end to end in the working section and giving a total plate length of 304.8 cm. A uniform wall temperature was achieved by heating the plates with steam, produced by boiling water in a partial vacuum in chambers beneath each of the plates. This method of heating is the same as that of Bell (1966), except that the plates were horizontal in this work. Wall to free-stream temperature differences so produced were typically 12°C . Thermocouples were embedded in the plate at regular intervals to check on uniformity of plate

temperature and as references for the measurement of temperature differences. The layer was tripped using a wire 0.5 mm in diameter taped to the plate surface.

Fluctuating quantities

Velocity fluctuations. Streamwise velocity fluctuations u were measured using a constant-temperature hot-wire anemometer of the type developed by Perry & Morrison at the University of Melbourne (see Perry & Morrison 1971*a*). Dynamic calibration techniques were used throughout (see Perry & Morrison 1971*b*; Morrison, Perry & Samuel 1972) and all wires were recalibrated after use as a check. If calibrations before and after use failed to agree to within $\pm 1\%$ the data were rejected. True r.m.s. values of the turbulence signals were determined by squaring and integrating the signal on an EAI TR-20 analog computer.

Temperature fluctuations. A constant-current hot-wire anemometer was used for the temperature fluctuation measurements. It was operated at very low resistance ratios (approximately 1.01), which for $3\mu\text{m}$ Wollaston† wire corresponds to a wire current of about 2 mA. (The anemometer was therefore very insensitive to velocity fluctuations.) The usual constant-current frequency compensation method was used. The compensator setting was adjusted using a signal generated by a fluctuating radiant heat source focused on the wire.

The variation of the wire time constant with age and with mean velocity was taken into account during all measurements. Wires were calibrated for temperature measurements by both static and dynamic methods to account for a suspected variation in the frequency response at low frequencies caused by unsteady heat conduction to or from the prongs.‡

Conditionally sampled measurements

A three-wire probe consisting of an X-array and an upstream temperature-sensitive wire was constructed to enable simultaneous extraction of streamwise (u) and normal (v) velocity fluctuations and temperature fluctuations (θ). The X-wires were calibrated by shaking the probe horizontally and vertically to determine their relative sensitivities to u and v in the manner described by Morrison *et al.* (1972). The instantaneous outputs were added and subtracted after appropriate scaling to separate the instantaneous u and v signals. The u , v and θ signals were then scaled and simple logical gating was used to separate vu and $v\theta$ into their respective quadrants and to determine various correlations as described later.

3. Results

Flow conditions

The streamwise variation of the static pressure coefficient $C_p = (p - p_0)/\frac{1}{2}\rho U_0^2$, where p_0 and $\frac{1}{2}\rho U_0^2$ are the reference static and dynamic pressures at the inlet to the working section, and of the surface temperature coefficient

† Throughout this work, platinum Wollaston wire was used.

‡ Full details of the measuring techniques, mean flow field results and tabulated data are available from the authors in the form of an internal report (FM7).

$C_T = (T_w - T_\infty)/(T_w - T_\infty)_R$ are shown in figure 1. T_∞ is the uniform free-stream temperature, T_w is the wall temperature and the suffix R refers to a reference station. A favourable pressure gradient occurred near the leading edge which appears to be associated with flow adjustment after contraction and was difficult to remove. Furthermore, a laminar boundary layer existed on the contraction wall upstream of the trip. The experiment of Bell (1966)† did not have these undesirable features since his plate was mounted on the tunnel centre-line and the leading edge was downstream of the contraction. It was found that the above features of this experiment contaminated the results on the first plate but the layer appears to have asymptoted to 'zero-pressure-gradient' behaviour on the second plate. Also, near the leading edge there was a defect in plate temperature, possibly caused by the locally high heat-transfer rates. This also occurred in the experiments of Bell.

Elimination of an unheated starting length

From the work of Reynolds *et al.* (1958) an unheated starting length has an effect on the temperature distribution within the layer. The authors wished to have a situation where the aerodynamic and thermal layers started from the same 'virtual origin' so that the data would not be contaminated by the effects of this unheated starting length.

This matching was achieved by a trial-and-error process of shifting the boundary-layer trip. Rotta (1962), who attributed the analysis to von Kármán, derived an expression for the variation of a non-dimensional momentum thickness with a non-dimensional streamwise distance. The analysis is a solution of the von Kármán momentum integral equation using the law of the wall and the velocity defect law. The authors carried out an analogous analysis for the development of the thermal layer and using the thermal integral equation obtained an expression for a non-dimensional enthalpy thickness in terms of a non-dimensional streamwise distance.

The origins of the layers were defined by the authors as those streamwise positions where the respective thicknesses go to zero. This corresponds very closely to where C_f' (the local skin-friction coefficient) and C_h' (the local Stanton number) go to infinity according to the mathematics. In spite of the favourable pressure gradient near the leading edge of the plate, the origins for the thermal and aerodynamic layers matched very well, unlike those in the work of Bell (1966). In his work there was an obvious mismatch and hence an unintended, unheated starting length.

Mean flow

Mean profiles of velocity and temperature were taken at five stations for a unit Reynolds number at the inlet to the working section of $1.22 \times 10^6 \text{ m}^{-1}$. The mean velocity profiles were found to correlate very well with the usual law of the wall and velocity defect law, except that the profiles taken at the first and second measuring stations showed an appreciable departure from the value of the Coles wake parameter of 0.55 suggested by Coles (1956).

† Data for this work are given in Coles & Hirst (1968, p. 282).

The mean temperature distribution followed a thermal law of the wall and thermal defect law satisfactorily. Local skin-friction coefficients and Stanton numbers were determined by use of a Clauser plot and a 'thermal Clauser plot' † respectively. Various values of parameters in the thermal law of the wall

$$\frac{\Theta}{\Theta_\tau} = \frac{\sigma_t}{\kappa} \log_e \frac{yU_\tau}{\nu} + A_H(\sigma)$$

were tested. Here $\Theta = T_w - T$, $\Theta_\tau = q_w / \rho c U_\tau$ (the friction temperature), q_w is the wall heat flux, ρ and ν are the fluid density and kinematic viscosity, σ the Prandtl number, σ_t the turbulent Prandtl number, c the specific heat and κ the von Kármán constant. $A_H(\sigma)$ is a Prandtl-number-dependent quantity. The values suggested by Kader & Yaglom (1972) gave the most satisfactory agreement with an overall heat balance on the second plate. These values (for air) are $\sigma_t = 0.85$ and $A_H = 3.8$ with $\kappa = 0.41$.

Fluctuating quantities

According to Townsend's (1956, p. 90) self-preserving flow hypothesis and the usual similarity scaling for turbulent boundary layers,

$$\frac{\tilde{u}}{U_\tau} = f_1\left(\frac{y}{\Delta}\right), \quad \frac{\tilde{\theta}}{\Theta_\tau} = f_2\left(\frac{y}{\Delta_T}\right),$$

where Δ is the Clauser thickness,

$$\Delta = \int_0^\infty \left(\frac{U_1 - U}{U_\tau}\right) dy,$$

Δ_T is an analogous thickness for the thermal layer defined by

$$\Delta_T = \int_0^\infty \left(\frac{\Theta_1 - \Theta}{\Theta_\tau}\right) dy,$$

a tilde denotes an r.m.s. value, U_1 is the free-stream velocity and $\Theta_1 = T_w - T_1$. This thickness differs from the aerodynamic thickness Δ apparently because of the breakdown of the Reynolds analogy. The molecular Prandtl number σ influences the heat conduction at the boundary through the viscous zone, thus causing the boundary conditions for heat transfer to differ from those for momentum transfer in the fully turbulent part of the flow. Furthermore, the 'turbulent Prandtl number' differs from unity.

The results shown in figure 2 indicate that there is a reasonable collapse for the downstream profiles, but the upstream profiles seem to be affected by the slight favourable pressure gradient present. (More so than the mean fields.) The authors do not believe that these results are in error since they were repeated many times with wires of different sizes and with different currents.

Close to the wall, the results should scale according to wall variables (figure 3) but show a systematic trend in the region just above the viscous zone. Perry & Abell (1975) have shown that, for a pipe, a region of overlap between an inner and outer law leads to $\tilde{u}/U_\tau = H$, where H is a universal constant. Thus, in a region

† Thermal Clauser charts involved as extra parameters C'_t and σ and were computer generated for each C'_t for $\sigma = 0.7$ (air).

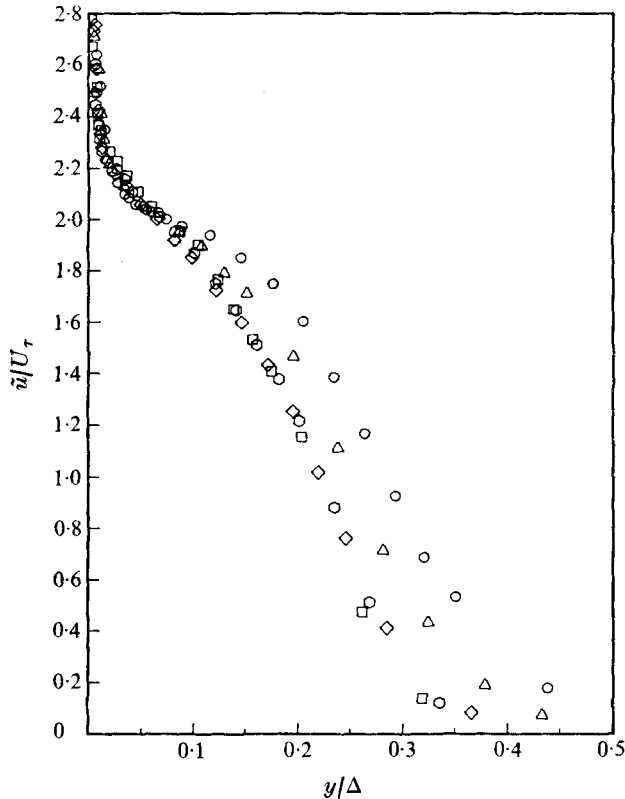


FIGURE 2 (a). For legend see facing page.

corresponding roughly to the logarithmic mean flow zone a constant \tilde{u}/U_τ should result. They showed that this implied that the energy-containing zone of the spectra scaled with U_τ and y alone. However, in a boundary layer, as distinct from fully developed pipe flow, the development length x could well be influencing the low wavenumber region, which may contain an appreciable amount of energy. Thus the scaling laws for the turbulence structure may be more complicated in boundary-layer flow. However, the results tend roughly to indicate the existence of an inner law (based on $y^+ = yU_\tau/\nu$) and an outer law for both velocity and temperature fluctuations. The \tilde{u}/U_τ results here show a 'knee' at a value of $\tilde{u}/U_\tau = 2.2$ as found by Perry & Abell in a pipe.

Instantaneous heat flux and Reynolds stress

In this subsection, various properties of the instantaneous Reynolds-stress and heat-flux quantities vu and $v\theta$ and correlations between them are examined.

The Reynolds analogy and its various modifications have been summarized by Squire (1953) and Kestin & Richardson (1963). These have been extended well beyond the original ideas of Reynolds (1874) and imply a very close similarity between vu and $v\theta$. The measurements reported here in fact show this, but they are by no means identical in behaviour.

Results were taken at the downstream measuring station ($R_x = 3.36 \times 10^6$) at

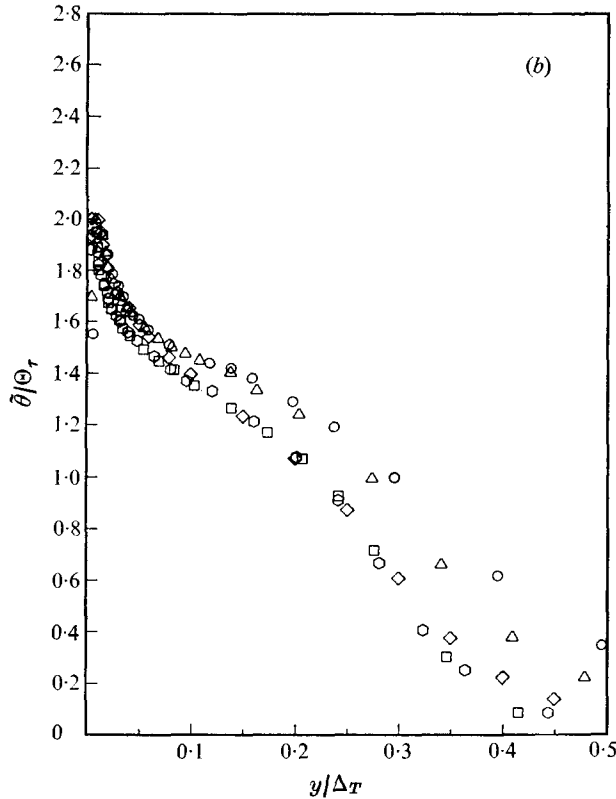


FIGURE 2. (a) \bar{u} intensities and (b) $\bar{\theta}$ intensities; outer-flow scaling.

	○	△	◇	◊	□
x (cm)	115	155	195	235	275
Δ (cm)	6.83	9.24	12.3	14.9	17.2
Δ_T (cm)	5.05	7.31	9.97	12.4	14.5

three wall distances corresponding to $y/\delta = 0.12, 0.30$ and 0.60 . The results were quite invariant with wall distance over the range investigated, so results for $y/\delta = 0.30$ are presented as being typical. Here δ is the 99% thickness.

The notation used is that the event vu being in quadrant i of the v, u plane is denoted by U_i and $v\theta$ being in quadrant j of the v, θ plane by Θ_j ($i, j = 1, 2, 3, 4$). The probability of occurrence of an event A is denoted by $P[A]$. In these planes v is taken as the ordinate while u and θ are the abscissae.

Occupancy time in each quadrant. The instantaneous fluxes vu and $v\theta$ were separated into quadrants and the durations as a fraction of the total time (or probability) of their presence in each quadrant were found to be

$$P[U_i] = \begin{bmatrix} 0.12 \\ 0.30 \\ 0.22 \\ 0.36 \end{bmatrix}, \quad P[\Theta_j] = \begin{bmatrix} 0.14 \\ 0.29 \\ 0.22 \\ 0.35 \end{bmatrix}.$$

The similarity between U_i and Θ_j is quite striking. However, more detailed correlations which follow show that the behaviour of the two fluxes is somewhat less similar.

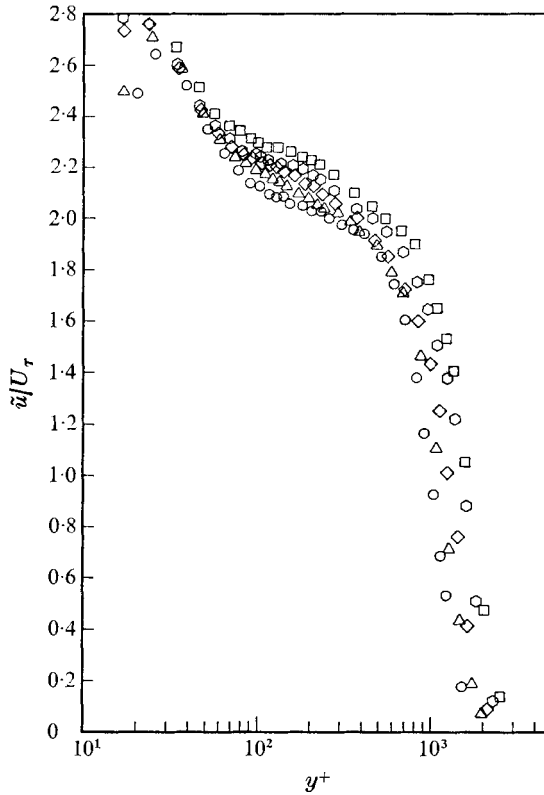


FIGURE 3 (a). For legend see facing page.

U_i, Θ_j correlations. The logical extension of the previous paragraph is to determine simultaneity of U_i and Θ_j . Denoting this by a matrix $\{Q_{ij}\}$, where

$$Q_{ij} = P[U_i \cap \Theta_j],$$

and labelling the rows by i and columns by j , the following results:

$$Q_{ij} = \begin{bmatrix} 0.08 & 0.06 & 0 & 0 \\ 0.05 & 0.24 & 0 & 0 \\ 0 & 0 & 0.14 & 0.08 \\ 0 & 0 & 0.08 & 0.27 \end{bmatrix}.$$

It can be seen that there are markedly high preferences for i and j coinciding (diagonal elements), particularly for $i = j = 2$ and 4 .

More significant is a conditional probability, for example the probability that U_i occurs given that Θ_j has occurred, denoted by

$$Q'_{ij} = P[U_i|\Theta_j] = P[U_i \cap \Theta_j]/P[\Theta_j].$$

It was found that

$$Q'_{ij} = \begin{bmatrix} 0.55 & 0.44 & 0 & 0 \\ 0.17 & 0.82 & 0 & 0 \\ 0 & 0 & 0.63 & 0.38 \\ 0 & 0 & 0.27 & 0.77 \end{bmatrix},$$

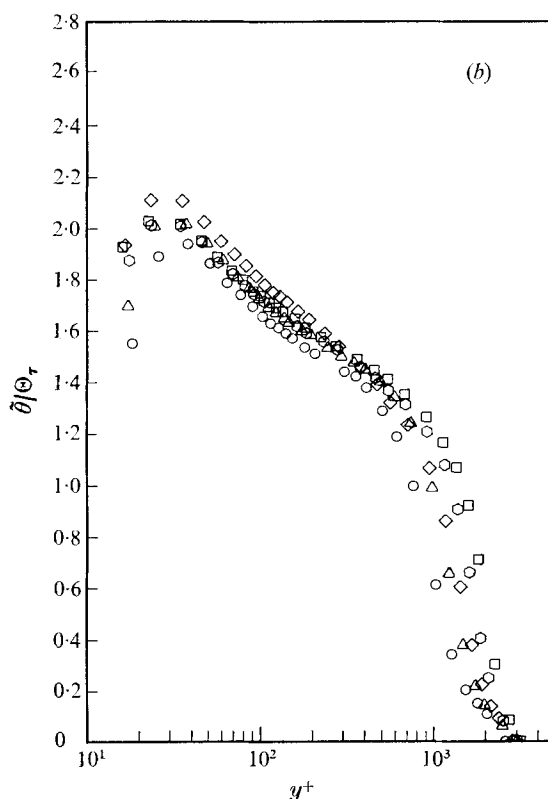


FIGURE 3. (a) \tilde{u} intensities and (b) $\tilde{\theta}$ intensities; inner-flow scaling. Symbols as in figure 2.

whereas for completely random events Q'_{ij} would be

$$Q'_{ij} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

The coincidence of events in a given quadrant is quite noticeable, again particularly for the second and fourth quadrants.

Investigation of behaviour with hole-size variations. Lu & Willmarth (1972) defined the 'hole size' H_u on the v, u plane by $|vu| = H_u \tilde{u}\tilde{v}$, where $|vu|$ is the modulus of the instantaneous value of vu and \tilde{u} and \tilde{v} are the r.m.s. values of u and v . An analogous hole size H_θ is now defined on the v, θ plane by $|v\theta| = H_\theta \tilde{v}\tilde{\theta}$.

The measurements of Lu & Willmarth show that most of the contribution to the Reynolds stress occurs outside a certain hole size. These contributions occur in a very intermittent manner and the instantaneous product vu appears as a spike on a cathode ray oscilloscope trace. When such spikes occur in the second quadrant the usual interpretation is that a burst has occurred, i.e. low velocity fluid is lifted away from the surface. If the spike occurs in the fourth quadrant a sweep is said to have occurred, i.e. high velocity fluid is 'diving' towards the

wall. The authors do not necessarily accept these physical pictures at this stage, but believe that these events (whatever they are) are probably associated with the outer bulges of the layer.

As the hole size is increased the frequency of occurrence of the event U_2 disappears more slowly than that of events in other quadrants, until for $H = 4-4.5$, only the frequency of occurrence of U_2 is non-zero. These events are said to be burst-related spikes and this hole size was used by Lu & Willmarth as a definition of when a burst occurred. Similarly, a hole size of $H_u = 2.25-2.75$ was used to define the occurrence of a sweep.

In this work, the mean values recommended by Lu & Willmarth were used: $H_u > 4.25$ in the second quadrant implies the occurrence of a burst and $H_u > 2.5$ in the fourth quadrant implies a sweep for both vu and (for want of better criteria) $v\theta$. However, since the correlation coefficients $\overline{vu}/\overline{v\tilde{u}}$ and $\overline{v\theta}/\overline{v\tilde{\theta}}$ have both been found to be approximately equal to 0.4 throughout the region of the layer examined it is expected that the hole sizes for vu and $v\theta$ would be the same.

Define the event \hat{U}_i as being $|vu|/\overline{v\tilde{u}} > H_u$ in the i th quadrant and $\hat{\Theta}_j$ as $|v\theta|/\overline{v\tilde{\theta}} > H_\theta$ in the j th quadrant. Then it is of interest to determine the probability of \hat{U}_i and $\hat{\Theta}_j$ occurring together given that, for example, $\hat{\Theta}_j$ has occurred, i.e.

$$R_{ij} = P[\hat{U}_i|\hat{\Theta}_j] = P[\hat{U}_i \cap \hat{\Theta}_j]/P[\hat{\Theta}_j].$$

The results obtained were

$$R_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.76 & 0 & 0 \\ 0 & 0 & 0.33 & 0 \\ 0 & 0 & 0 & 0.29 \end{bmatrix} \quad \text{for } H = 2.5,$$

$$R_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{for } H = 4.25.$$

The results show very high correlations between burst and sweep events for vu and $v\theta$. Of particular note is the fact that, given a burst event for $v\theta$, there is a probability of $\frac{3}{4}$ of a burst occurring in vu .

Mean time intervals between bursts and sweeps. Rao *et al.* (1971) suggested that the mean time intervals between bursts (\overline{T}_B) and sweeps (\overline{T}_S) scale according to an 'outer-flow' time scale δ^*/U_1 , where δ^* is the displacement thickness and U_1 is the free-stream velocity. The results are presented in figure 4 for vu and $v\theta$ as a function of hole size H . These results agree well with those of Lu & Willmarth, as indicated on the figure, and show that the process is an inviscid one. This strongly suggests that the bursts and sweeps are associated with the motion of the outer bulges.

The characteristic mean times between bursts and between sweeps of vu were found to be about 32 and 40 respectively, compared with the results of Lu & Willmarth of 32 and 30. For $v\theta$, the characteristic mean times between bursts and between sweeps were found to be 75 and 30. These differences may well be due to an incorrectly chosen hole size for $v\theta$, and more work on this is needed.

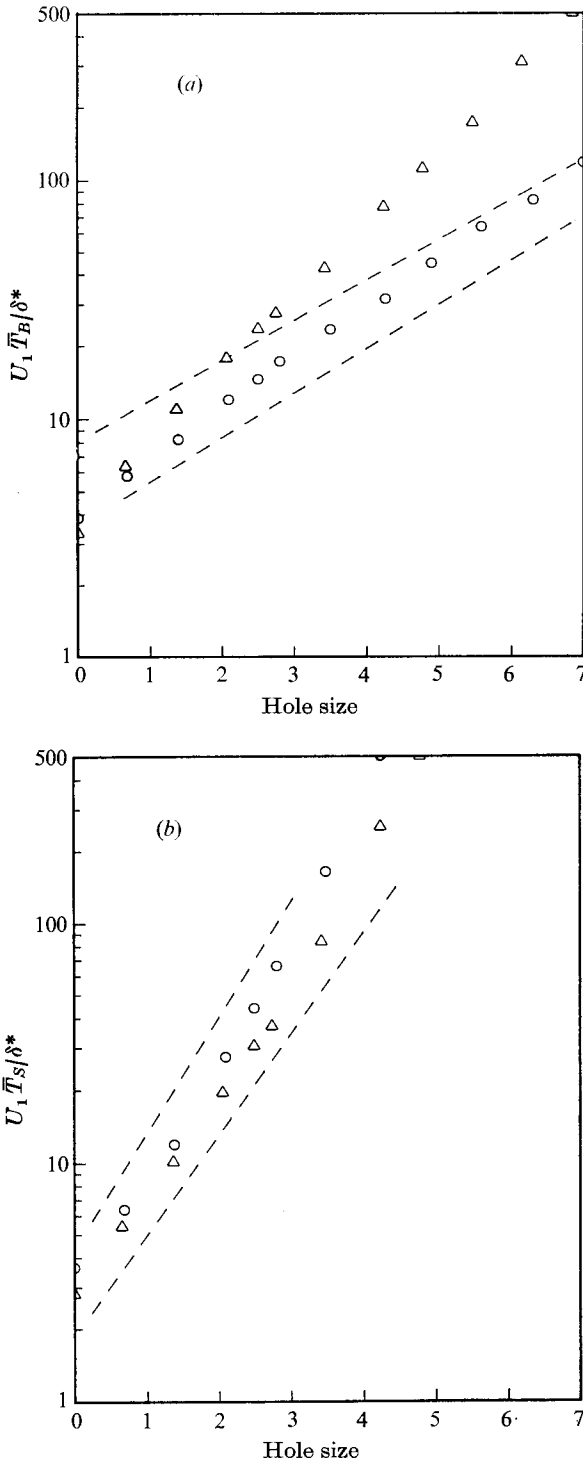


FIGURE 4. Outer-flow scaling for mean time interval between (a) bursts and (b) sweeps. \circ , vu ; \triangle , $v\theta$; —, spread of results of Willmarth & Lu.

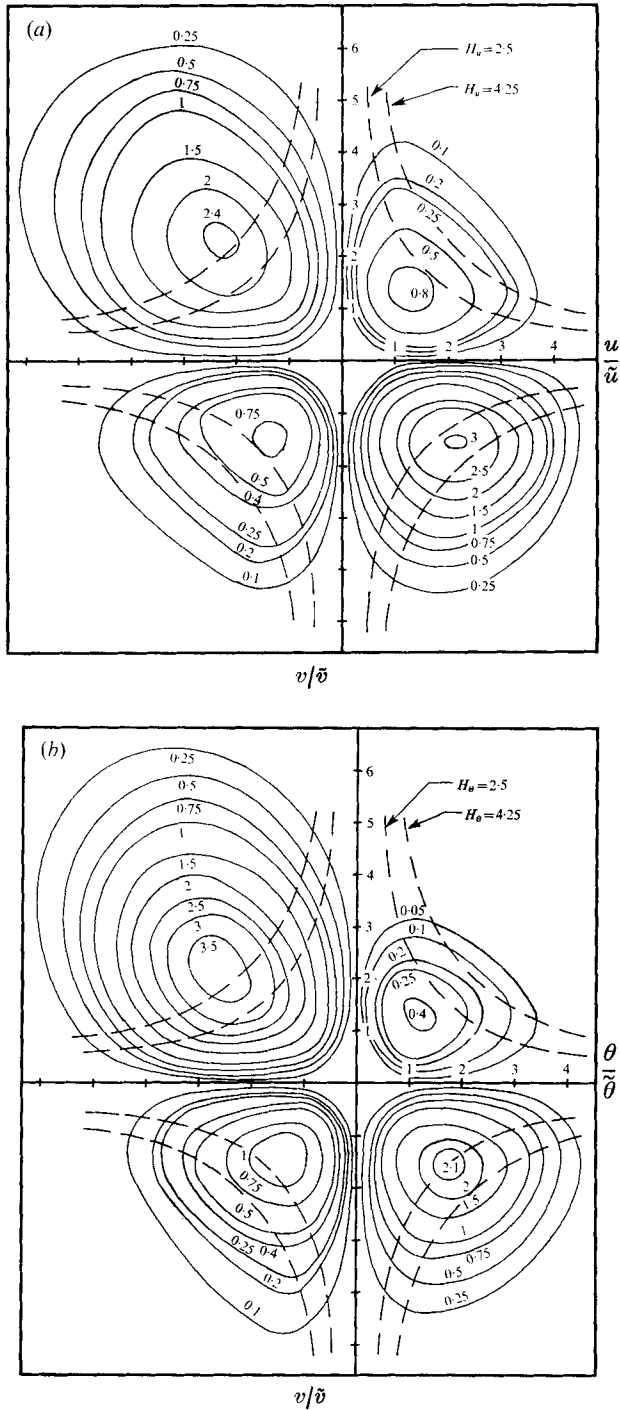


FIGURE 5. Weighted joint probability density distribution of (a) vu and (b) $v\theta$. Arbitrary contour values, same scaling throughout.

vu		$v\theta$	
Quadrant	Contribution	Quadrant	Contribution
1	-0.08 (-0.105)	1	-0.03
2	+0.75 (+0.805)	2	+0.97
3	-0.11 (-0.125)	3	-0.10
4	+0.44 (+0.425)	4	+0.16

TABLE 1. Contribution to vu and $v\theta$ from each quadrant

Joint probability density distributions. Weighted joint probability density distributions of v and u and v and θ were measured, with weighting according to the instantaneous Reynolds stress or heat flux. The contours are thus defined by

$$\frac{vu}{\bar{v}\bar{u}} p\left(\frac{v}{\bar{v}}, \frac{u}{\bar{u}}\right) = \text{constant}, \quad \frac{v\theta}{\bar{v}\bar{\theta}} p\left(\frac{v}{\bar{v}}, \frac{\theta}{\bar{\theta}}\right) = \text{constant},$$

where p is the joint probability density function defined such that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{vu}{\bar{v}\bar{u}} p\left(\frac{v}{\bar{v}}, \frac{u}{\bar{u}}\right) d\left(\frac{v}{\bar{v}}\right) d\left(\frac{u}{\bar{u}}\right) = \frac{\overline{vu}}{\bar{v}\bar{u}}.$$

The results are shown in figure 5. Also shown are the hyperbolas $|vu|, |v\theta| = \text{constant}$, defining hole sizes of 2.5 and 4.25. As mentioned earlier, the results of Lu & Willmarth (1972) indicate that, for a hole size of approximately 4.25, only in the second quadrant is there significant activity outside the hole, but this is not so for the weighted distributions on the v, u and v, θ planes. The values assigned to the contours are proportional to $(vu/\bar{v}\bar{u}) p(v/\bar{v}, u/\bar{u})$ and $(v\theta/\bar{v}\bar{\theta}) p(v/\bar{v}, \theta/\bar{\theta})$. The scale factor is arbitrary but is the same for both planes so that relative magnitudes of Reynolds-stress and heat-flux contributions can be directly compared.

The contribution of each quadrant to the Reynolds stress or heat flux was determined, and is shown in table 1. The bracketed values are the results of Lu & Willmarth taken at $y^+ = 30$.

Again, the similarity between the behaviour of vu and $v\theta$ is evident.

4. Conclusions and discussion

Mean flow profiles of velocity and temperature appear to correlate well with an inner and outer scaling with a region of overlap. Suitable matching conditions have been established to ensure that the origins of the thermal and aerodynamic layers are the same. However, the outer length scales for these two layers are different.

The turbulence structure of velocity and temperature appears to be much more sensitive to spurious pressure gradients than are the mean flow profiles. The results do, however, indicate that, given sufficient flow development, an inner and outer scaling similar to that of the mean flow results is roughly applicable.

Comparisons of the various conditionally sampled results for Reynolds heat flux and Reynolds stress show that the behaviour of these quantities is strikingly similar. 'Sweeps' and 'bursts' in Reynolds stress are accompanied to a significant degree by equivalent occurrences in the heat flux.

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